AQA

Please write clearly in block capitals.

Centre number						Candidate number				

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

AS MODEL SOLUTIONS
FURTHER MATHEMATICS
Paper 2 – Mechanics

Thursday 17 May 2018 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Answer **all** questions in the spaces provided.

1 A particle A, of mass 0.2 kg, collides with a particle B, of mass 0.3 kg

Immediately before the collision, the velocity of A is $\begin{bmatrix} 4 \\ 12 \end{bmatrix}$ m s^{-1}

and the velocity of B is $\begin{bmatrix} -1 \\ -3 \end{bmatrix}$ m s^{-1}

As a result of the collision the particles coalesce to become a single particle.

Find the velocity of the single particle.

Circle your answer.

[1 mark]

$\begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix}$ m s^{-1} $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ m s^{-1} $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ m s^{-1} $\begin{bmatrix} 3 \\ 9 \end{bmatrix}$ m s^{-1}

Conservation of momentum:

$0.2\begin{pmatrix} 4 \\ 12 \end{pmatrix} + 0.3\begin{pmatrix} -1 \\ -3 \end{pmatrix} = 0.5\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$0.4\begin{pmatrix} 4 \\ 12 \end{pmatrix} + 0.6\begin{pmatrix} -1 \\ -3 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ ms^{-1}

2 A train is travelling at maximum speed with its engine using its maximum power of 1800 kW

When travelling at this speed the train experiences a total resistive force of 40 000 N

Find the maximum speed of the train.

Circle your answer.

[1 mark]

22 m s^{-1} 45 m s^{-1} 54 m s^{-1} 90 m s^{-1}

$P = Fv$

(Power = Driving Force x velocity)

$\frac{P}{F} = v \Rightarrow \frac{1800 \times 10^3}{40 \times 10^3} = 45$ ms^{-1}

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3 The kinetic energy, E, of a compound pendulum is given by

$$E = \frac{1}{2} I \omega^2$$

where ω is the angular speed and I is a quantity called the moment of inertia.

3 (a) Show that for this formula to be dimensionally consistent then I must have dimensions ML^2, where M represents mass and L represents length.

[2 marks]

$KE = \frac{1}{2}mv^2 \Rightarrow [KE] = M(LT^{-1})^2 = ML^2T^{-2}$

So $[E] = ML^2T^{-2}$

$[\omega] = T^{-1}$

$\left(I = \frac{2E}{\omega^2}\right)$

$[I] = \frac{ML^2T^{-2}}{(T^{-1})^2} = ML^2$

3 (b) The time, T, taken for one complete swing of a pendulum is thought to depend on its moment of inertia, I, its weight, W, and the distance, h, of the centre of mass of the pendulum from the point of suspension.

The formula being proposed is

$$T = kI^{\alpha}W^{\beta}h^{\gamma}$$

where k is a dimensionless constant.

Determine the values of α, β and γ.

[3 marks]

$[I^{\alpha}W^{\beta}h^{\gamma}] = (ML^2)^{\alpha}(MLT^{-2})^{\beta}(L)^{\gamma}$

$= M^{\alpha}L^{2\alpha}M^{\beta}L^{\beta}T^{-2\beta}L^{\gamma}$

$= M^{\alpha+\beta}L^{2\alpha+\beta}T^{-2\beta}$

$\alpha + \beta = 0$

$-2\beta = 1 \Rightarrow \beta = -\frac{1}{2}, \alpha = \frac{1}{2}$

$2\alpha + \beta + \gamma = 0 \Rightarrow \gamma = \frac{1}{2}$

Turn over ▶

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4 Two smooth spheres A and B of equal radius are free to move on a smooth horizontal surface.

The masses of A and B are m and $4m$ respectively.

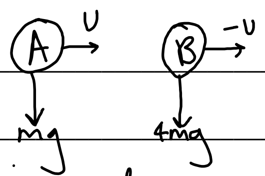
The coefficient of restitution between the spheres is e.

The spheres are projected directly towards each other, each with speed u, and subsequently collide.

4 (a) Show that the speed of B immediately after the impact with A is

$$\frac{u(3-2e)}{5}$$

[4 marks]



Conservation of momentum:

$4mu - mu = mv_A + 4mv_B$

$3u = v_A + 4v_B$ ①

Newton's Law of Restitution:

$e = \frac{v_A - v_B}{u - -u}$

$\Rightarrow e = \frac{v_A - v_B}{2u}$

$2ue = v_A - v_B$

$v_A = 2ue + v_B$ ②

Sub ② into ①

$2ue + v_B + 4v_B = 3u$

$5v_B = 3u - 2ue$

$v_B = \frac{u(3-2e)}{5}$

4 (b) Find the speed of A in terms of u and e.

[2 marks]

$v_A = v_B + 2ue$

$v_A = \frac{u(3-2e)}{5} + 2ue$

$v_A = \frac{3u - 2eu + 10eu}{5}$

$v_A = \frac{8eu + 3u}{5}$

4 (c) Comment on the direction of motion of the spheres after the collision, justifying your answer.

[2 marks]

As $0 \le e \le 1$, both V_A and V_B are positive
B will be following A, travelling in the same direction

4 (d) The magnitude of the impulse on B due to the collision is I.

Deduce that

$$\frac{8mu}{5} \le I \le \frac{16mu}{5}$$

[3 marks]

$I = 4m(v_B - U_B)$

$= 4m\left(\frac{u(3-2e)}{5} - u\right)$

$= 4m\left(\frac{3u - 2eu - 5u}{5}\right)$

$= \frac{4m(-2u(e+1))}{5}$

$I = \frac{-8mu}{5}(1+e)$

$|I| = \frac{8mu}{5}(1+e)$

As $0 \le e \le 1$, if $e = 0$: $|I| = \frac{8mu}{5}$

if $e = 1$: $|I| = \frac{16mu}{5}$

so $\frac{8mu}{5} \le I \le \frac{16mu}{5}$

5 A car travels around a roundabout at a constant speed. The surface of the roundabout is horizontal.

The car has mass 990 kg and the path of the car is a circular arc of radius 48 metres.

A simple model assumes that the car is a particle and the only horizontal force acting on it as it travels around the roundabout is friction.

On a dry day typical values of friction, F, between the surface of the roundabout and the tyres of the car are

$$7300\,\text{N} \leq F \leq 9200\,\text{N}$$

5 (a) Using this model calculate a safe speed limit, **in miles per hour**, for the car as it travels around the roundabout.

Explain your reasoning fully.

Note that there are 1600 metres in one mile.

[4 marks]

$F = ma \quad a = \frac{v^2}{r} \Rightarrow F = \frac{mv^2}{r}$

Friction = force

$7300 = \frac{mv^2}{r}$

$v = \sqrt{7300\left(\frac{48}{990}\right)}$ ms^{-1}

$v = 42$ mph

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5 (b) Gary assumes that on a wet day typical values for friction, F, are

$$5400\,\text{N} \leq F \leq 10\,000\,\text{N}$$

Comment on the validity of Gary's revised assumption.

[2 marks]

Wet conditions reduce friction so it could always b ≤ 9200.

It couldn't be 10,000 as 10,000 > 9200

Turn over for the next question

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6 At a fairground a dodgem car is moving in a straight horizontal line towards a side wall that is perpendicular to the velocity of the car.

The speed of the car is $1.8\,\text{m s}^{-1}$

It collides with the side wall and rebounds along its original path with a speed of $1.2\,\text{m s}^{-1}$

The total mass of the dodgem car and the passengers is 250 kg

6 (a) Find the magnitude of the impulse on the car during the collision with the side wall.

[2 marks]

$I = \Delta p = m(\Delta v) = m(v - u)$

$= 250(1.2 - -1.8)$

$= 250(3)$

$= 750 \text{ kg ms}^{-1}$

6 (b) A possible model for the magnitude of the force, F newtons, acting on the dodgem car due to its collision with the side wall is given by

$$F = kt(4 - 5t) \qquad \text{for } 0 \leq t \leq 0.8$$

6 (b) (i) Find the value of k.

[3 marks]

$750 = \int_0^{0.8} kt(4-5t)\,dt$

$\int_0^{0.8} t(4-5t)\,dt = \left[2t^2 - \frac{5}{3}t^3\right]_0^{0.8} = 2(0.8)^2 - \frac{5}{3}(0.8)^3$

$= \frac{32}{75}$

So $750 = \frac{32}{75}k$

$\Rightarrow k = 1800$ (2sf)

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6 (b) (ii) Determine the maximum magnitude of the force predicted by the model.

[2 marks]

$\frac{d}{dt} t(4-5t) = \frac{d}{dt} 4t - 5t^2 = 4 - 10t$

Derivative $= 0 \Rightarrow$ Stationary point

$4 - 10t = 0 \Rightarrow t = \frac{4}{10} = 0.4s$

$\frac{d^2}{dt^2}(4t - 5t^2) = -10 < 0$ so $4(t-5t)$ is a maximum at $t = 0.4s$

$I = kt(4-5t) = \frac{750(75)}{32}(0.4)(4-5(0.4)) = 1400$ N (2sf)

Turn over for the next question

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7 **Use g as 9.8 m s^{-2} in this question.**

Dominic, a bungee jumper of mass 75 kg, has his ankles attached to one end of a cord. The other end of the cord is attached to a bridge which is 50 metres above the surface of a river.

The cord can be modelled as a light elastic cord of natural length 25 metres and modulus of elasticity 3200 N. Dominic is modelled as a particle.

Dominic steps off the bridge at the point where the cord is attached and falls vertically downwards.

7 (a) Find Dominic's speed at the point when the cord initially becomes taut.

[2 marks]

	Before	After
GPE	$75g(25)$	0
KE	0	$\frac{1}{2}(75)v^2$

GPE lost = KE gained

$75g(25) = \frac{1}{2}(75)v^2$

$2g(25) = v^2$

$v^2 = 50g$

$v = 22 ms^{-1}$ (2sf)

7 (b) Determine whether or not Dominic enters the river and gets wet.

[5 marks]

	Before	After
EPE	0	$\frac{\lambda x^2}{2l}$
PE	mgh	0

PE lost = EPE gained

$\frac{\lambda x^2}{2l} = 75(9.8)(25+x)$

$\frac{3200x^2}{2(25)} = 75(9.8)(25+x)$

$3200x^2 = 50(75)(9.8)(25) + 50(75)(9.8)x$

$64x^2 = 18375 + 735x$

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$64x^2 - 735x - 18375 = 0$

$x = 23.6$ m

$23.6 + 25 = 48.6$ m so Dominic does not get wet

7 (c) One limitation of this model is that Dominic is not a particle.

Explain the effect of revising this assumption on your answer to part **(b)**.

[2 marks]

Dominic has a height which a particle does not so his distance descended would be larger than 48.6m

Dominic might get wet if he was 1.4 m taller

END OF QUESTIONS

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